Computing Slow Optical Flow By Interpolated Quadratic Surface Matching

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Abstract

Many approaches to estimate velocity field (optical flow) from sequential images have been proposed since the 70's. The major previous work can typically be divided into two branches: gradient-based methods and correlation-based methods (involving frequency-domain, template matching, etc.). Here, we propose a new technique considering both of the thoughts to realize a highly accurate estimation of slow optical flow (all of velocities under $1\ pixel/frame$). The concept is to interpolate quadratic surfaces of brightness intensity in the vicinity of each pixel on successive frames, then under the rigidness assumption, the brightness and its gradient distribution of a local region on each frame are matched, the local velocity vector then can be calculated with an iteration algorithm. Experimental result shows the effectiveness of proposed method by using several synthetic and real images.

1. Introduction

Optical flow computation, i.e., estimation of velocity field from sequential images, is one of the most important techniques in machine vision. For at least two decades of research, there have been many ideas given to solve problems of estimation accuracy or implementation efficiency. In this paper, we concentrate on the former, with an expectation to the amazing development of performance of computers.

Classic methods of optical flow computation can be classified to two branches typically: gradient-based methods [8, 9, 10] and correlation-based methods [1, 5, 7]. The former uses the relationship between temporal change and spatial gradient of intensity of each pixel to derive velocity vector, therefore it is possible to obtain 100% density optical flow of the entire image. As the measurements perform numerical differentiations, smoothness constraints to temporal-spatial changes are needed, and the estimation accuracy often becomes bad when noise occurs or the intensity changes are extremely big or small (e.g. aperture problem). The later, correlation-based methods, except spatio-temporal frequency correlation analysis, compare regions of usually large extent in the successive images to find the displacement between them. The defect of region-based method's qualities is that the density of estimation is not sufficient, and usually displacement vectors at prominent points or contour lines are estimated only.

To estimate the optical flow which is slower than $1.0\ pixel/frame$ (p/f), gradient-based methods are adequate, but the techniques need to compute numerical differentiation–an extremely unstable operation. Meanwhile, using some local time-space windows or patches, e.g., linear filters [6], least-squares(LS) [2], quadtree splines [11], instantaneous short range flow (subpixel motion) can be calculated efficiently. In this paper, we propose a method which combines both the thoughts of gradient-based class and region-based class, to estimate optical flow with high accuracy and density, especially for the velocity fields which are slower than $1.0; p/f$). Our approach is to consider the intensity distribution of a local small region can be approximated by a quadratic surface, then according to the rigidness assumption, the best matching can be performed between the interpolated region in 2 or 3 frames. The optical flow is computed by minimizing the matching error if we consider the displacement of the regions as the velocity values. Adopting spatial smoothness constraint, the estimation accuracy of optical flow is improved efficiently, and experimental result shows less estimation errors than previous differential techniques.

2. Formulation

The purpose of this study is to detect a slow optical flow in an apparent motion, which we define it as a low velocity field where the entire displacements of images are under 1.0 p/f.

2.1. Interpolating quadratic surface

Let the brightness at a point (pixel) (x, y) on frame t of image sequence be denoted by $I_t(x, y)$, and the distribution of the brightness be $g_t(x, y)$, then to the central area of the region,

$$I_t(x, y) \approx g_t(x, y). \qquad (1)$$

We consider $g_t(x, y)$, the brightness distribution in the small region, can be represented by a quadratic surface

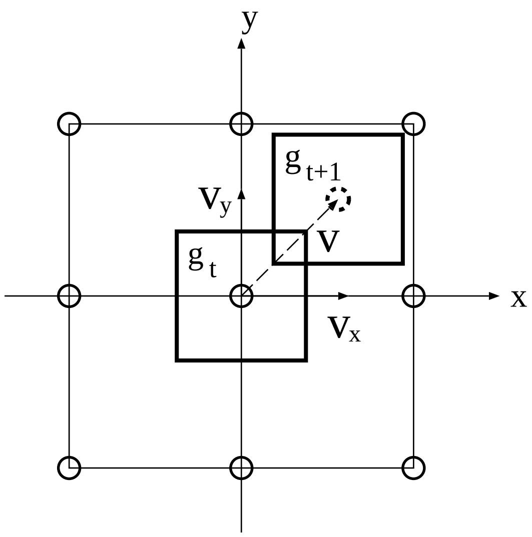


Figure 1. Interpolated regions: g_t and g_{t+1} are matched to calculate the object displacement $V(v_x, v_y)$ between successive images.

$$g_t(x, y) = A_t x^2 + B_t xy + C_t y^2 + D_t x + E_t y + F_t, \quad (2)$$

then the coefficients $A_t, B_t, C_t, D_t, E_t, F_t$ can be calculated by using the values of adjacent points (pixels) (Fig. 1). i.e.,

$$Minimize \sum_{x=-1}^{1} \sum_{y=-1}^{1} \left[I_t(x, y) - g_t(x, y)\right]^2. \qquad (3)$$

and the solution should be

$$\begin{aligned}
A_t = \;& \tfrac{1}{6}(I_t(-1,-1) + I_t(-1,0) + I_t(-1,1) + I_t(1,-1) \\
& + I_t(1,0) + I_t(1,1) - 2I_t(0,-1) - 2I_t(0,0) \\
& - 2I_t(0,1)) \\
B_t = \;& \tfrac{1}{4}(I_t(-1,-1) + I_t(1,1) - I_t(-1,1) - I_t(1,-1)) \\
C_t = \;& \tfrac{1}{6}(I_t(-1,-1) + I_t(0,-1) + I_t(1,-1) + I_t(-1,1) \\
& + I_t(0,1) + I_t(1,1) - 2I_t(-1,0) - 2I_t(0,0) \\
& - 2I_t(1,0)) \\
D_t = \;& \tfrac{1}{6}(I_t(1,-1) + I_t(1,0) + I_t(1,1) - I_t(-1,-1) \\
& - I_t(-1,0) - I_t(-1,1)) \\
E_t = \;& \tfrac{1}{6}(I_t(-1,1) + I_t(0,1) + I_t(1,1) - I_t(-1,-1) \\
& - I_t(0,-1) - I_t(1,-1)) \\
F_t = \;& \tfrac{1}{9}(5I_t(0,0) + 2I_t(-1,0) + 2I_t(1,0) + 2I_t(0,-1) \\
& + 2I_t(0,1) - I_t(-1,-1) - I_t(1,-1) - I_t(-1,1) \\
& - I_t(1,1)).
\end{aligned} \qquad (4)$$

The computation of the coefficients can be expressed by 6 operators shown in Appendix.

2.2. Computation of optical flow

Assuming that the brightness and the form of objects in images do not change temporally,and the local flow velocity is $V(v_x, v_y)$, we have

$$g_t(x, y) = g_{t+\Delta}(x + \Delta v_x, y + \Delta v_y), \qquad (5)$$

where Δ is the delay of time, can be unity to deal with the low velocity field.

Then (v_x, v_y) can be derived by minimizing

$$H(v_x, v_y) = \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} \left[g_{t+1}(x+v_x, y+v_y) - g_t(x, y)\right]^2 dxdy, \qquad (6)$$

where $0 < \alpha, \beta < 1$ (2 small squares of g_t and g_{t+1} in Fig 1). In fact, we use Newton-Raphson iteration as

$$v_x^{k+1} = v_x^k - \frac{H_{yy}H_x - H_{xy}H_y}{H_{xx}H_{yy} - H_{xy}H_{yx}}, \qquad (7)$$

$$v_y^{k+1} = v_y^k - \frac{H_{xx}H_y - H_{yx}H_x}{H_{xx}H_{yy} - H_{xy}H_{yx}}, \qquad (8)$$

where k is the step of iteration, H_x, H_y and H_{xx}, H_{xy}, H_{yy} are the first-order partial derivatives and second-order partial derivatives of $H(v_x, v_y)$, respectively. These derivatives are able to be derived from Eq. 2 and calculated explicitly by the intesities of 9 pixels. The local brightness matching, Eq. 6, is similar to the approach of first-order derivatives of Horn & Schunck [8], i.e. *optical flow formula* $\frac{\partial I(x,y,t)}{\partial x}v_x + \frac{\partial I(x,y,t)}{\partial y}v_y + \frac{\partial I(x,y,t)}{\partial t} = 0$. To consider the spatio-temporal gradient of the quadratic surface, a high order matching can be expressed by

$$H^*(v_x, v_y) = \lambda_1 H(v_x, v_y) + \lambda_2 \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} \Big[\frac{\partial g_{t+1}(x+v_x, y+v_y)}{\partial x} - \frac{\partial g_t(x,y)}{\partial x}\Big]^2 + \Big[\frac{\partial g_{t+1}(x+v_x, y+v_y)}{\partial y} - \frac{\partial g_t(x,y)}{\partial y}\Big]^2) dxdy, \tag{9}$$

where $H^*(v_x, v_y)$ is the total error of matching, $\lambda_1, \lambda_2 > 0$ is the regularization parameters(Only 1 parameter is enough, in fact. Here we set up 2 for the convenience of experiment statement). This matching of local brightness gradient is similar to the thought of second-order derivatives of Nagel [9, 10].

Furthermore, to avoid aperture-problem, the additional constraint of velocity's spatial smoothness can be adopted, the matching error function Eq. 9 then is advanced to

$$H^{**}(v_x, v_y) = H^*(v_x, v_y) + \lambda_3^2 S(v_x, v_y). \tag{10}$$

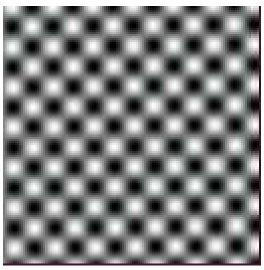
Here, parameter $\lambda_3 > 0$,

$$S(v_x, v_y) = \big(\bar{v_x} - v_x\big)^2 + \big(\bar{v_y} - v_y\big)^2, \tag{11}$$

where $\bar{v_x}, \bar{v_y}$ is the average velocity of points nearby v_x, v_y, respectively.

3. Experiments

Synthetic and real images are used to compare proposed technique with classic optical flow estimation methods. Because our method provides 100% density of flow, the gradient-based methods (Horn & Schunck [8], Nagel [9, 10]) are compared in experiments. The experiment is named as *Proposed 1* when λ_2 in Eq. 10 is zero, where brightness matching and spatial smoothness constraint are calculated [12]. The experiment is named as *Proposed 2* when λ_1, λ_2, and $\lambda_3 > 0$, that means brightness matching, brightness gradient matching and spatial smoothness constraint are calculated [13]. To confirm the basic estimation accuracy of these algorithms, noisy images are not considered here.

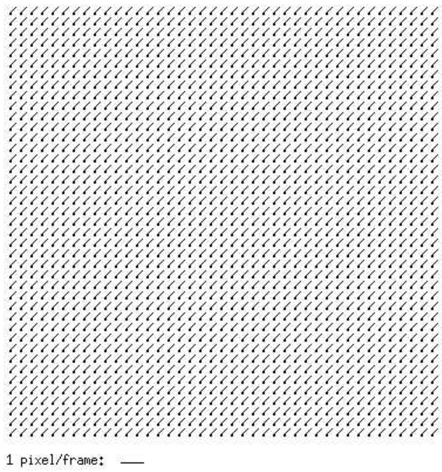


128x128,256 levels

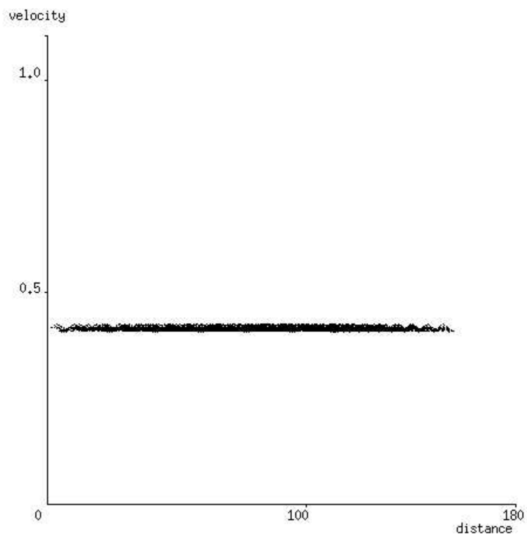
Figure 2. A frame of synthetic patterns

3.1. Synthetic image

Fig. 2 shows a frame of synthetic image sequence. The real value of the flow is $(v_x = 0.3 \;\; p/f, v_y = 0.3 \;\; p/f, i.e., |V(v_x, v_y)| = 0.42 \;\; p/f)$ on all points in the image. We applied Eq. 6 which using the constraint of brightness only, to compute the velocity field of the images, and obtained a rough optical flow. Eq. 9 which adopts gradient of intensities, gave a progress of estimation accuracy. Considering the spatial smoothness constraint of velocity, i.e. Eq. 10, more accurately estimates are obtained which average error is less than previous works (Table 1). Fig. 3(a) shows the estimated flow, where parameters are



(a) $\lambda_1, \lambda_2, \lambda_3$: 1.0, 1.0, 70.0



(b) $\lambda_1, \lambda_2, \lambda_3$: 1.0, 1.0, 70.0

Figure 3. Estimated velocities

$\lambda_1 = \lambda_2 = 1.0, \lambda_3 = 70.0, \alpha = \beta = 0.5$, in the case after 1000 iterations. Fig. 3(b) gives a visual judgment which horizontal axis expresses distance from the point to the origin (0,0) (on the top of up-left), and vertical axis expresses the velocity value of the point. In this simulation, all points should be on a line where velocities are equal to $0.42\ p/f$.

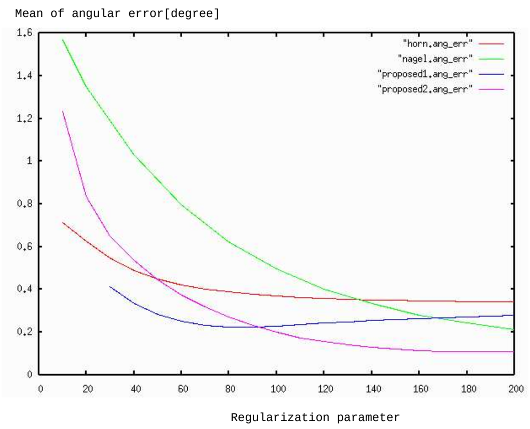


Figure 4. Comparison of average of estimation error

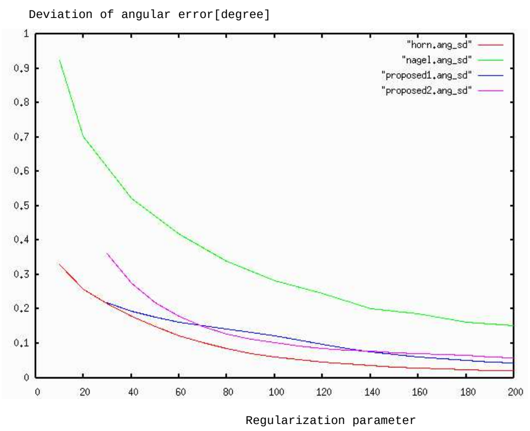


Figure 5. Comparison of standard deviation of estimation error

Fig. 4 and Fig. 5 shows comparisons of estimation errors (mean and deviation, respectively), by classical gradient methods and proposed method via the different values of regularization parameters. Errors are measured by applying the error analysis method of Barron [3], the angle between the true and estimated motions expresses the error level. All the estimation errors decreased when the values of regularization parameters increased, but lower means of average errors (degree) show the proposed method is more effective to the slow velocity field, i.e., under $1.0\ p/f$ (Fig. 6).

Table 1 shows the estimation errors with optimum values of parameters in experiment, proposed method approached

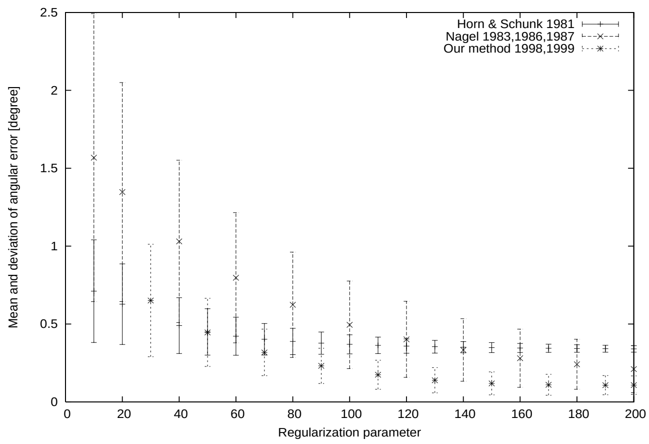


Figure 6. Estimation errors via regularization parameters

higher accuracy than classic gradient-based techniques (About $50 \sim 70\%$ average, and $-200 \sim 60\%$ standard deviation of degree errors are reduced). We also tried some

Table 1. Error comparison

	Param. value	Ave. err.($^\circ$)	Std. dev.($^\circ$)
Horn[8]	200	0.34	0.02
Nagel[10]	200, 10000	0.21	0.15
Proposed 1	1, 0, 90	0.22	0.13
Proposed 2	1, 1, 190	0.11	0.06

rotation flows, and the proposed method showed effective either.

3.2. Real image

Fig. 7(a) gives a frame of a real image sequence, recorded using a CCD camera, which moves to the right. The slowest velocity is $0.2; p/f$ approximately, where is the wall at the rear of the vase. The high velocities are less than $0.5; p/f$, where are the areas of vase and the front of the floor. Fig. 8(a)∼(d) shows the estimated flows with different values of parameters. Fig. 8(a) is resulted by using brightness matching only(Eq. 6). Fig. 8(b) used brightness gradient matching only (Eq. 9 without $H(v_x, v_y)$, and $\lambda_2 = 1$). Fig. 8(c) was calculated by Eq. 9, without spatial smoothness constraint ($\lambda_3 = 0$). Fig. 8(d) shows the result of estimation by Eq. 10, where $\lambda_1 = 1.0, \lambda_2 = 1.0, \lambda_3 = 70.0$(*Proposed 2*). The estimated data(Fig. 8(d)) are divided into 3 levels of low velocities, middle velocities and high velocities. Fig. 7(b) shows the levels in grey intensity. Depth map transformed from estimated optical flow, which can be applied to 3-D reconstruction of ob-

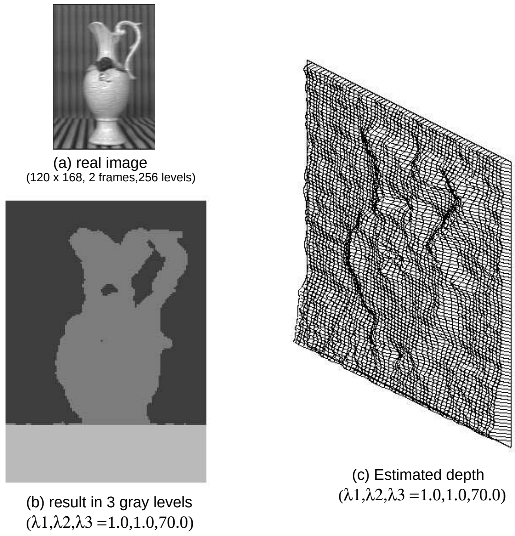


Figure 7. Real image and the estimation result

jects, are expressed by Fig. 7(c). To evaluate the accuracy of estimation, we compared the standard deviation of velocities on areas of wall, vase and floor (3 grey regions in Fig. 7(b)). Our proposed method showed lower statistical variance($3 \sim 6\%$), which means higher estimation accuracy than those computed by the classic methods(see Fig. 9 and Table 2) .

Table 2. Standard deviation of velocities on 3 grey areas in Fig. 7(b)

	Param. Value	Wall	Vase	Floor
Horn[8]	100	0.032	0.016	0.048
Nagel[10]	100, 10000	0.058	0.017	0.043
Proposed 1	1, 0, 100	0.040	0.019	0.050
Proposed 2	1, 1, 190	0.031	0.015	0.045

4. Conclusion

According to the algorithm that minimize the difference between a tamplate and an image, image alignment or image registration have been implemented, and applied to optical flow estimation by many researches [5, 11, 7]. In this paper, we proposed a new technique of optical flow computation by interpolated quadratic surface matching. First-order derivatives (matching of brightness) and second-order derivatives (matching of brightness gradient) were consider in the technique. The proposed technique can estimate instantaneous dense optical flow only using 2 or 3

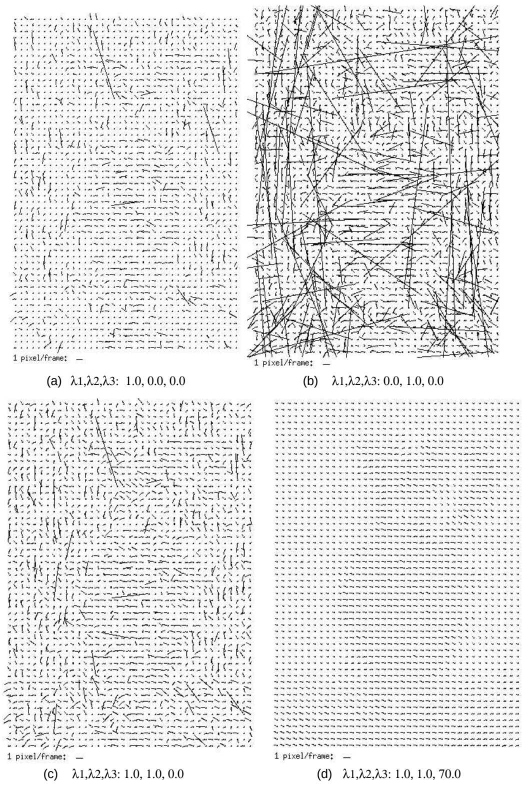


Figure 8. Results with different values of parameters

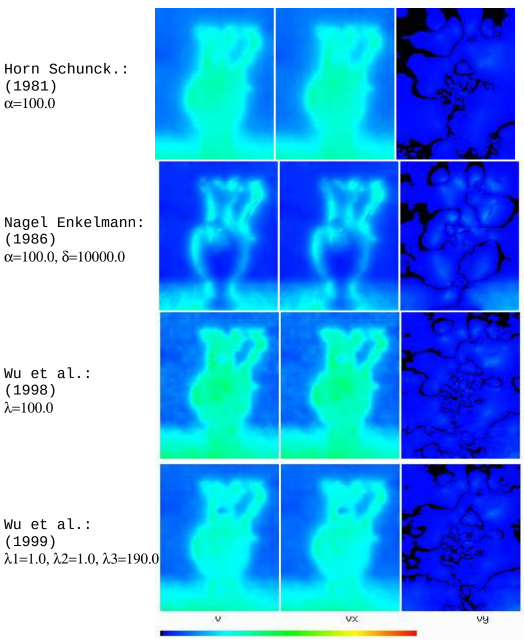


Figure 9. Results of different methods

frames, especially, for sub-pixel motion. The experimental comparison shows its high performance especially to estimate slow velocity field under $1.0\ p/f$, and the technique is effective to real image sequence.

We used 9 (3 x 3) pixels to obtain the 6 parameters of general quadratic surface function here, in fact, 6 pixel's data is enough. The remainders are expected be used to deal with noise or discontinuities. And there are lots of improvement ideas like hierarchical-estimation, repetition-matching, multi-frame matching, noise elimination process, regularization of parameters, etc., are considerable to be adapted to this interpolated quadratic surface matching method.

Acknowledgments

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Appendix

Fig.10 shows the coefficients of quadratic surface function, which are given by 3x3 operators according to Eq. 3 in Chapter 2.

$$At : \frac{1}{6}\begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 1 \\ 1 & -2 & 1 \end{bmatrix} \qquad Bt : \frac{1}{4}\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$Ct : \frac{1}{6}\begin{bmatrix} 1 & 1 & 1 \\ -2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix} \qquad Dt : \frac{1}{6}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$Et : \frac{1}{6}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \qquad Ft : \frac{1}{9}\begin{bmatrix} -1 & 2 & -1 \\ 2 & 5 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

Figure 10. Operators for the coefficients of quadratic surface function

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